Implementing an interval computation library for OCaml on x86/AMD64 architectures

J-M. Alliot   J-B. Gotteland   **C. Vanaret**   N. Durand   D. Gianazza

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1. Introduction
   - Objectives
   - Interval arithmetic
   - Why a new interval library?

2. A functional implementation for OCaml
   - Unexpected problems
   - Modules
   - Implementation principles

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Objectives

Bound global solutions of **difficult continuous optimization** problems

$$\begin{aligned} \min_{\mathbf{x} \in \mathcal{D}} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & g_i(\mathbf{x}) \leq 0, \quad i \in \{1, \dots, p\} \\ & h_j(\mathbf{x}) = 0, \quad j \in \{1, \dots, q\} \end{aligned}$$

Hybridization of optimization methods [ADGG12]

- **Evolutionary Algorithms** (EA)
- **Interval Branch & Bound** algorithm (BB)
  computation of lower/upper bound of f over a subspace of $\mathcal{D}$

Interval arithmetic

Numerical analysis method to bound round-off errors [Moo66]

Problem $(\mathcal{P})$

- $f(x, y) = 333.75y^6 + x^2(11x^2y^2 - y^6 - 121y^4 - 2) + 5.5y8 + x/(2y)$
- $f(77617, 33096) = -0.827396$ (6 digits)
- OCaml 3.12 compiler: $-1.180592.10^{21}$

Interval arithmetic

Interval arithmetic

- Extends to intervals $\{+, -, *, /\}$, $\exp$, $\log$, etc.
- An **interval extension** F of f yields a rigorous enclosure of $f(X)$
- Computation of lower/upper bound with outward rounding

$[a, b] \oplus [c, d] = [a +_{low} c, b +_{high} d]$

- $a +_{low} c$ must be a lower bound of $a + c$
- $b +_{high} d$ must be an upper bound of $b + d$

Problem $(\mathcal{P})$
Interval arithmetic yields: $[-5.902958.10^{21}, 5.902958.10^{21}]$

Interval arithmetic

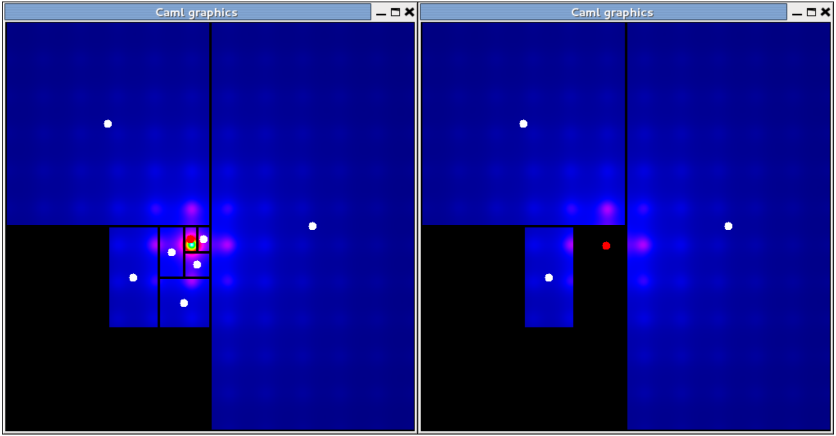
F interval extension

- $f(x, y) = x^2 + \exp(y), x \in [-3, 2], y \in [0, 1]$:

$$\begin{aligned} F([-3, 2], [0, 1]) &= [-3, 2]^2 + \exp([0, 1]) \\ &= [0, 9] + [1, e] = [1, e + 9] \end{aligned}$$

- $f(x) = \sin(x), x \in [1, 2]$: $F([1, 2]) = [\sin(1), 1]$

Interval Branch and Bound Algorithms



Why a new interval library?

Existing interval libraries

- C++ libraries: PROFIL/BIAS, Interval in Boost C++, Gaol
- Sun interval library: Fortran 95 or C++ [Mic01]
- MPFI: C or C++ [RR02]
- ...

Motivations for a new library

- Preference for OCaml functional programming
- Need for speed and efficiency

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Unexpected problems

C (and OCaml) math functions

- do not return the same results on 32- and 64-bit architectures
- may yield wrong results in low and high rounding modes

Necessity to write **low-level functions** in assembly language (restricted to x86/Amd64 architectures)

A functional implementation for OCaml

Modules

- **Chcw**: elementary float functions
  - in all rounding modes (nearest, low, high)
  - written in C and assembly language
- **Fpu**: OCaml binding to Chcw
- **Interval**: OCaml interval arithmetic
- **Fpu_rename**: redefinition of OCaml float functions
  - Except float operators: `(+.)` `(-.)` `(*.)` `(/.)`
- **Fpu_rename_all**: redefinition of all OCaml float functions
  - Including float operators

Module Fpu

```
val ffloat: int -> float
val ffloat_high: int -> float
val ffloat_low: int -> float
(** Float functions. The float function is exact on 32-bit machines
but not on 64-bit machines with ints larger than 53 bits *)

val fadd: float -> float -> float
val fadd_low: float -> float -> float
val fadd_high: float -> float -> float
(** Floating-point addition in nearest, low and high modes *)

val fsub: float -> float -> float
val fsub_low: float -> float -> float
val fsub_high: float -> float -> float
(** Floating-point substraction in nearest, low and high modes *)
```

Module Fpu_rename_all

```
val (+.): float -> float -> float
val (-.): float -> float -> float
val (/.): float -> float -> float
val ( *.): float -> float -> float

val mod_float: float -> float -> float
val sqrt: float -> float
val log: float -> float
val exp: float -> float

val ( ** ): float -> float -> float
```

- Opening the **Fpu_rename_all** module ensures that these functions return the same results on all architectures

Interface of module Interval

```
type interval = {
   low: float; (** lower bound *)
   high: float (** upper bound *)
}
   (...)
val pi_I: interval
   (...)
val float_i: int -> interval
   (...)
val (+$.): interval -> float -> interval
val (+$): interval -> interval -> interval
   (...)
val sqrt_I: interval -> interval
val pow_I_i: interval -> int -> interval
```

Implementation principles

Rules

- No "empty" interval (raise an exception instead)
- Interval bounds may be infinite
- Interval bounds are not supposed to be **nan**
  Lower bound is not supposed to be **infinity**
  Upper bound is not supposed to be **neg_infinity**
- **nan**, **infinity** and **neg_infinity** operands are not handled

Examples

- $1/[0, 0]$ fails
- $1/[0, 1]$ returns $[1, +\infty]$
- $1/[-1, 1]$ returns $[-\infty, +\infty]$

Module Interval

```
(...)
let inv_I {low = a; high = b} =
  let sa = compare a 0. and sb = compare b 0. in
  if sa = 0 then
    if sb = 0 then failwith "inv_I"
    else {low = fdiv_low 1. b; high = infinity}
  else if 0 < sa || sb < 0 then {low = fdiv_low 1. b; high = fdiv_high 1. a}
  else if sb = 0 then {low = neg_infinity; high = fdiv_high 1. a}
  else {low = neg_infinity; high = infinity}
(...)
```

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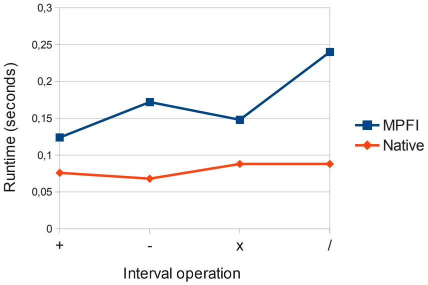
Performance comparison

Comparison with an OCaml binding to MPFI
(runtime for 10^6 operations)



Conclusion

Native implementation

- low-level redefinition of elementary functions
- reliable, fast despite the functional paradigm
- available under GNU Lesser General Public License

Successfully used in our hybrid optimization algorithm

- computation of optima of Michalewicz function (improvement for deterministic methods)
- applications to aeronautical problems (air traffic conflict resolution)

References I

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Questions

For implementation details, please contact

Jean-Marc Alliot
jean-marc.alliot@irit.fr

Jean-Baptiste Gotteland
gottelan@cena.fr

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ENAC

Institut de Recherche en Informatique de Toulouse
CNRS INPT UPS UT1

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