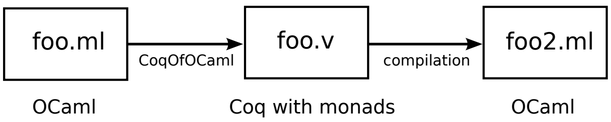
Coq of OCaml

OCaml Workshop 2014

1 Introduction

The *CoqOfOCaml* project is a compiler from a subset of the *OCaml* language to the *Coq* programming language. This compiler aims to allow reasoning about *OCaml* programs, or to import existing *OCaml* libraries in *Coq*.

Chain of compilation The *CoqOfOCaml* compiler imports an *OCaml* program `foo.ml` into a *Coq* file `foo.v`. We encode effects using a monadic translation [2], since *Coq* allows only purely functional and terminating functions, mimicking the way the *Haskell* language handles imperative traits. These effects can be run within *Coq*, although very slowly. Reversely, we can compile `foo.v` into an *OCaml* file `foo2.ml`. This compilation removes all the proof terms and translates encoded effects to concrete imperative *OCaml* effects.



Specification The specification of our tool is twofolds. First, we require that `foo2.ml` behaves as `foo.ml` (has the same input-output traces). Second, we require that `foo2.ml` behaves as `foo.v` when we interpret the effects in *Coq*. Therefore the user can continue to work on and update `foo.v` once the original program is imported. This is the case of the development process in which the user imports `foo.ml` just once at the beginning, and only works on `foo.v` afterward.

Effects system Until now, we focused on the import process, not on the proof techniques on *Coq* programs. The import is based on a type and effects system [1]. We compose an effects inference with a monadic translation guided by the effects. We chose not to encode effects in one big generic monad but in many fine grained and composable ones. We hope it leads to simpler specifications and proofs, localizing effects only where they are needed. We rely on the dependent types to represent these composable effects in *Coq*.

Related work Other tools have been developed to formally verify functional programs with effects in *Coq*, like *CFML*, *Ynot* or *Why*. Our contribution is novel because we generate a shallow embedding in *Coq* of an existing higher-order language (*OCaml*), with the ability to extract it back to an equivalent program with effects.

We will present our effects system, explain how our implementation works, show some case studies and conclude with future directions.

2 Design

We will describe the effects system, how effects are inferred in *OCaml* and how effectful terms are represented in *Coq*.

Effects For this first version, we restricted ourselves to a simple effects system with no polymorphic effects, keeping that part as a future work.

An *effects descriptor* is an unordered set of atomic descriptors, acting as names for exceptions, top-level references or the special input-output descriptor:

$$d := \{a_1, \dots, a_n\}$$

An *effect type* is the shape of an *OCaml* type with additional effects information. The syntax is the following:

$$\begin{array}{rcl} \tau & := & \text{Pure} \\ & | & \text{Pure} \xrightarrow{d} \tau \end{array}$$

So an expression can be pure (as effect-free and terminating) or a function generating the descriptor d when applied. In particular arguments of a function have to be effects-free.

Types and effects inference in *OCaml* We first infer the datatypes using the *OCaml* compiler's front-end. Then we derive the effects of each sub-expression in a bottom-up analysis with an environment of effects. For mutually recursive functions, we compute a fixpoint until convergence.

Representation in *Coq* We use a monadic encoding to represent effects in *Coq*. Given a type A and monad M, an expression e of type $M\ A$ is a computation returning a value of type A. To compose computations there are two basic operators:

$$\begin{array}{l} \text{return} : \forall A, A \to M\ A \\ \text{bind} : \forall A\ B, M\ A \to (A \to M\ B) \to M\ B \end{array}$$

We associate a monad M_d to each descriptor d, definable in *Coq* thanks to dependent types. For a d given by $\{a_1, \dots, a_n\}$:

$$M_d\ A = (S_1 \times \cdots \times S_n) \to (A + E_1 + \cdots + E_n) \times (S_1 \times \cdots \times S_n)$$

where S_i is the mutable state of a_i and E_i the error which can be raised by a_i. We chose an arbitrary order to index the atomic descriptors a_i, knowing that there is a canonical isomorphism between two definitions given two different orders. This monad is composable: M_{d_1} and M_{d_2} compose into $M_{d_1 \cup d_2}$. Besides, this composition is commutative, in opposition to the composition of monad transformers.

To combine different computations with different descriptors we use the lift operator. If $d \subseteq d'$:

$$\text{lift}_{d,d'} : \forall A, M_d\ A \to M_{d'}\ A$$

Two different computations of descriptors d_1 and d_2 can be lifted to two computations of descriptor $d_1 \cup d_2$ and composed with the standard bind operator of $M_{d_1 \cup d_2}$.

The monads for a reference of type S and an exception carrying a value of type E (with $\emptyset$ the empty type) are defined as:

$$\begin{array}{rcl} M_{\{\mathrm{Ref}_S\}}\,A & = & S \to (A + \emptyset) \times S \\ M_{\{\mathrm{Exn}_E\}}\,A & = & \mathrm{unit} \to (A + E) \times \mathrm{unit} \end{array}$$

On the contrary, effects like the non-termination are encoded using references and exceptions. This effect is used to represent non-terminating functions or functions whose termination is not proven. The non-termination is the composition of a reference to a decreasing counter (the fuel) and of an exception raised on non-terminated computations (when the counter reaches zero).

3 Implementation

The *CoqOfOCaml* compiler is implemented in *OCaml*. It imports a subset of the *OCaml* abstract syntax tree typed by the *OCaml* compiler front-end. Then it infers effects, does the monadic translation and pretty-print the output in the *Coq* syntax.

We support the pure lambda-calculus kernel, mutually recursive definitions, inductive types definitions with pattern-matching, records, abstract types and modules. We do not handle signatures and functors. We have exceptions and global references. We support the main parts of the `Pervasives` and `List` libraries.

This compiler was challenging since we needed to import a large subset of *OCaml* to start working on real programs. We decided not to support functors due to code complexity. We may be able to reduce this code complexity using the new annotation mechanism of *OCaml* to directly annotate nodes of the syntax tree with effects.

A lot of care was given to get an output both readable and close to what a real programmer could write. Indeed, the user is supposed to work on the *Coq* version once the *OCaml* code had been imported.

4 Case studies

We successfully imported the slightly modified `List`, `Set` and `Map` modules from the standard *OCaml* library. These modules work on immutable structures but are not purely functional: they contain exceptions and functions whose termination is not obvious.

Here is an example of the `map2` function as defined in the *OCaml* library:

```
let rec map2 f l1 l2 =
  match (l1, l2) with
    ([], []) -> []
  | (a1::l1, a2::l2) ->
    let r = f a1 a2 in r :: map2 f l1 l2
  | (_, _) -> invalid_arg "List.map2"
[@@coq_rec]
```

This is the imported version:

```
Fixpoint map2 {A B C : Type} (f : A -> B -> C)
  (l1 : list A) (l2 : list B)
  : M [ OCaml.Invalid_argument ] (list C) :=
  match (l1, l2) with
  | ([], []) => return []
  | (cons a1 l1, cons a2 l2) =>
    let r := f a1 a2 in
    let! x := map2 f l1 l2 in
    return (cons r x)
  | (_, _) =>
    OCaml.Pervasives.invalid_arg "List.map2" % string
  end.
```

The `[@@coq_rec]` annotation in *OCaml* tells the compiler to trust the `Fixpoint` operator of *Coq* to validate the termination instead of using the non-termination monad. The return type in *Coq* explicitly mentions that `map2` may raise the `OCaml.Invalid_argument` exception. The monadic `bind` is noted `let!`.

Here is a comparison of the number of lines of code in the original *OCaml* files and the generated *Coq* ones:

Module	Lines of *OCaml*	Lines of *Coq*	Increase
List	396	622	+57%
Set	349	539	+54%
Map	310	497	+60%

The size increase was mainly due to the monadic translation, especially in the case of non-termination where we define an auxiliary function. We hope this size increase is small enough so the user can continue to work easily on the imported *Coq* files.

We have not worked on the reasoning rules about monadic programs yet. Still, it was possible to manually make a proof in *Coq* about an absence of exception in the generated code: in the `List` module, the `sort` function depends on an auxiliary function `chop` which may raise an exception `Assert_failure`. Its importation into *Coq* is:

```
Fixpoint chop {A : Type} (k : Z) (l : list A)
  : M [ OCaml.Assert_failure ] (list A) := ...
```

It removes the first `k` elements of a list `l`, and fails if there is no element left to remove. The precondition $0 \leq k \leq \mathrm{length}\, l$ ensures that `chop` will succeed. We added this precondition in *Coq* and proved that the failing branch is never reached. Hence, we transformed `chop` into an effect-free function:

```
Fixpoint chop {A : Type} (k : Z) (l : list A)
  {struct l} : 0 <= k -> k <= length l ->
  { l' : list A | length l' = length l - k }.
```

Using this new `chop` function, we updated the definition of `sort` and proved that it never raises an exception either.

5 Conclusion

We have presented a tool, *CoqOfOCaml*, which can import existing examples of *OCaml* programs to *Coq* and extract them to *OCaml*. An effects system and a monadic translation is used to represent effectful programs in *Coq*. The sources can be downloaded on `https://github.com/clarus/coq-of-ocaml`.

In the future we would like to investigate more the programming and proof techniques on *Coq* programs with effects. Some interesting problems are the extension of the effects system to handle the polymorphism, an implementation of an effects inference mechanism on *Coq* terms, the representation of new effects (including concurrency), the design of reasoning rules on monadic programs with dependent types, and the certification of an extraction chain to *OCaml* with effects.

References

[1] Lucassen and Gifford. Polymorphic effect systems. 1988.

[2] Philip Wadler. The essence of functional programming. 1992.